Exercise 10C: Solutions

1 $2\pi - a,\ 2\pi - b,\ 2\pi - c,\ 2\pi - d$

2 a $\sin x = \dfrac{-\sqrt{3}}{2}$

$\therefore x = \dfrac{4\pi}{3},\ \dfrac{5\pi}{3}$ as $x \in [0,\ 2\pi]$

b

$\sin(2x) = -\dfrac{\sqrt{3}}{2},\ x \in [0,\ 2\pi]$

$\therefore 2x \in [0,\ 4\pi]$

$\therefore 2x = \dfrac{4\pi}{3},\ \dfrac{5\pi}{3},\ 2\pi + \dfrac{4\pi}{3},\ 2\pi + \dfrac{5\pi}{3}$

as $2x \in [0,\ 4\pi]$

$\therefore x = \dfrac{2\pi}{3},\ \dfrac{5\pi}{6},\ \dfrac{5\pi}{3},\ \dfrac{11\pi}{6}$ as $x \in [0,\ 2\pi]$

c

$2\cos 2x = -1$

$\therefore \cos 2x = -\dfrac{1}{2},\ x \in [0,\ 2\pi]$

$\therefore 2x \in [0,\ 4\pi]$

$\therefore 2x = \dfrac{2\pi}{3},\ \dfrac{4\pi}{3},\ 2\pi + \dfrac{2\pi}{3},\ 2\pi + \dfrac{4\pi}{3}$

as $2x \in [0,\ 4\pi]$

$\therefore x = \dfrac{\pi}{3},\ \dfrac{2\pi}{3},\ \dfrac{4\pi}{3},\ \dfrac{5\pi}{3}$

as $x \in [0,\ 2\pi]$

d

$\sin\left(x + \dfrac{\pi}{3}\right) = -\dfrac{1}{2},\ x \in [0,\ 2\pi]$

$\therefore x + \dfrac{\pi}{3} \in \left[\dfrac{\pi}{3},\ \dfrac{7\pi}{3}\right]$

$\therefore x + \dfrac{\pi}{3} = \dfrac{7\pi}{6},\ \dfrac{11\pi}{6}$

as $x + \dfrac{\pi}{3} \in \left[\dfrac{\pi}{3},\ \dfrac{7\pi}{3}\right]$

$\therefore x = \dfrac{5\pi}{6},\ \dfrac{3\pi}{2}$ as $x \in [0,\ 2\pi]$

e

$2\cos\left(2\left(x+\frac{\pi}{3}\right)\right)=-1$

$\therefore \cos\left(2\left(x+\frac{\pi}{3}\right)\right)=-\frac{1}{2},\ x\in[0,\ 2\pi]$

$\therefore x+\frac{\pi}{3}\in\left[\frac{\pi}{3},\ \frac{7\pi}{3}\right]$

$\therefore 2\left(x+\frac{\pi}{3}\right)\in\left[\frac{2\pi}{3},\ \frac{14\pi}{3}\right]$

$\therefore 2\left(x+\frac{\pi}{3}\right)=\frac{2\pi}{3},\ \frac{4\pi}{3},\ 2\pi+\frac{2\pi}{3},$

$2\pi+\frac{4\pi}{3},\ 4\pi+\frac{2\pi}{3}$

as $2\left(x+\frac{\pi}{3}\right)\in\left[\frac{2\pi}{3},\ \frac{14\pi}{3}\right]$

$\therefore x+\frac{\pi}{3}=\frac{\pi}{3},\ \frac{2\pi}{3},\ \frac{4\pi}{3},\ \frac{5\pi}{3},\ \frac{7\pi}{3}$

$\therefore x=0,\ \frac{\pi}{3},\ \pi,\ \frac{4\pi}{3},\ 2\pi$ as $x\in[0,2\pi]$

f $2\sin\left(2x+\frac{\pi}{3}\right)=-\sqrt{3}$

$\therefore \sin\left(2x+\frac{\pi}{3}\right)=\frac{-\sqrt{3}}{2},\ x\in[0,\ 2\pi]$

$\therefore 2x\ \in[0,\ 4\pi]$

$\therefore\ 2x+\frac{\pi}{3}\in\left[\frac{\pi}{3},\ \frac{13\pi}{3}\right]$

$\therefore 2x+\frac{\pi}{3}=\frac{4\pi}{3},\ \frac{5\pi}{3},\ 2\pi+\frac{4\pi}{3},\ 2\pi+\frac{5\pi}{3}$

as $2x+\frac{\pi}{3}\in\left[\frac{\pi}{3},\ \frac{13\pi}{3}\right]$

$\therefore 2x=\pi,\ \frac{4\pi}{3},\ 3\pi,\ \frac{10\pi}{3}$

$\therefore x=\frac{\pi}{2},\ \frac{2\pi}{3},\ \frac{3\pi}{2},\ \frac{5\pi}{3}$ as $x\in[0,\ 2\pi]$

3 a $-\frac{5\pi}{6},-\frac{\pi}{6}$

b $0,-\frac{2\pi}{3},-\frac{\pi}{3},\frac{\pi}{3},\frac{2\pi}{3},\pi,-\pi$

c 0

d $0,-\frac{2\pi}{3}$

e $-\frac{5\pi}{6},-\frac{\pi}{2},\frac{\pi}{6},\frac{\pi}{2}$